

UMBB / IGEE

Set # 1

EE 172 / LO 2

Exo 1:

a/ Find the domain and the range of the following functions

1/ $f(x, y) = \frac{e^x - e^y}{e^x + e^y}$, 2/ $f(x, y) = \frac{1}{\sqrt{y - x^2}}$

3/ $f(x, y, z) = \frac{z}{x - y}$, 4/ $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 1}}$

b/ Find and sketch the domain of $f(x, y) = \frac{x^2 + y^2}{x^2 - y^2}$ Exo 2:

Find all of the first order partial derivatives for the following functions

1/ $f(x, y) = x^4 + 6\sqrt{y} - 10$

2/ $f(x, y, z) = x^2 y - 10 y^2 z^3 + 43x - 7 \tan(4y)$

3/ $f(x, y) = x^7 \ln(y^2) + \frac{9}{x^3} - \sqrt[7]{y^4}$

Exo 3:

a/ Find all the second order derivatives for

$f(x, y) = \cos(2x) - x^2 e^{5y} + 3y^2$

b/ Find f_{xxxyzz} for $f(x, y, z) = \frac{z^2}{3y} \ln x$ c/ Find $\frac{\partial^3 f}{\partial y \partial x^2}$ for $f(x, y) = e^{xy}$ Exo 4:Given $f(x, y) = x e^{-x^2 y^2}$, show that $f_{xy}(x, y) = f_{yx}(x, y)$ Exo 5:Show that the given f satisfies the corresponding partial differential equation

a/ $u = \frac{x^2 y^2}{x + y}$; $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u$

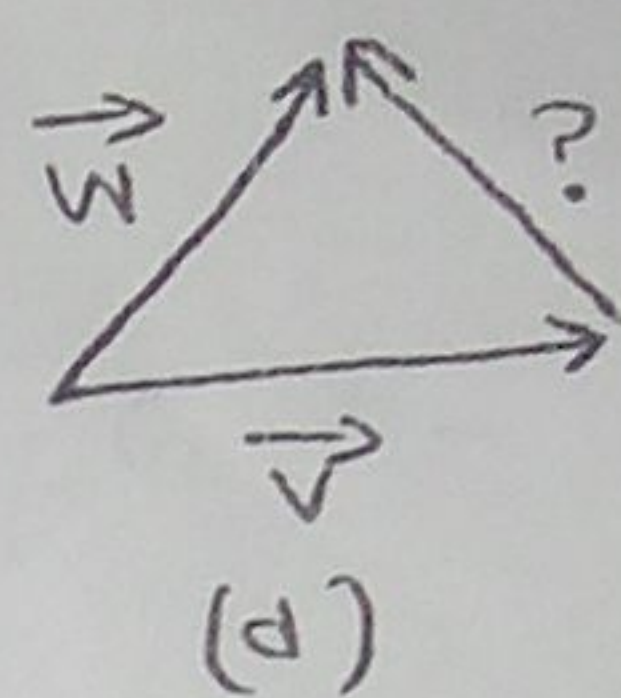
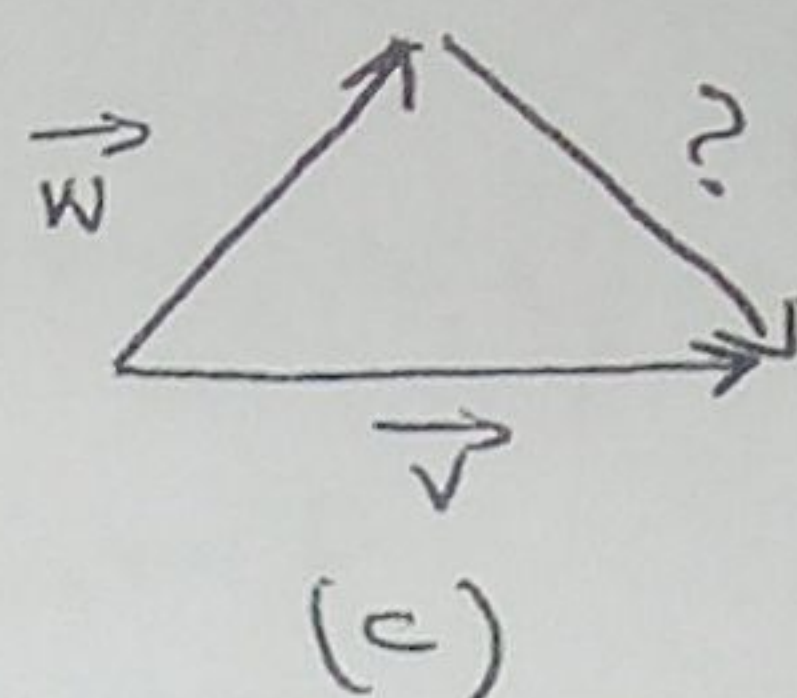
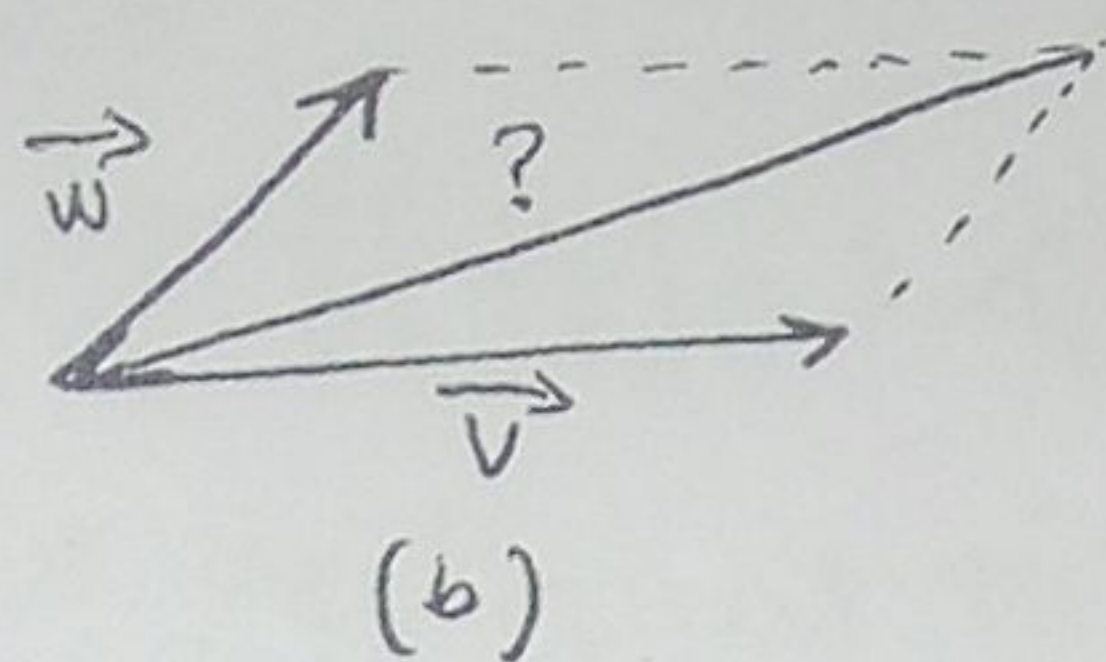
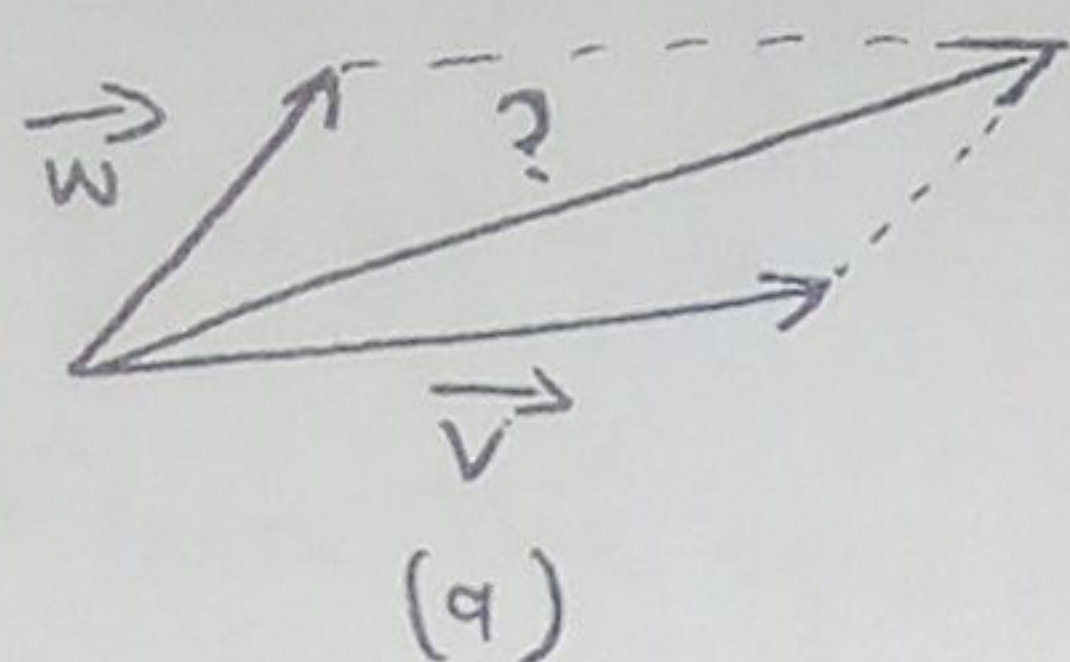
b/ $u = x^2 y + y^2 z + z^2 x$; $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2$

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Set # 2

Exo 1: Label the vectors:



Exo 2: Calculate

a/ $(\vec{i} + \vec{j} + \vec{k}) \times (2\vec{i} + \vec{k})$, b/ $[(\vec{i} - \vec{j}) \times (\vec{j} - \vec{k})] \times (\vec{i} + 5\vec{k})$
c/ $\vec{i} \times \vec{j}$, d/ $\vec{j} \times \vec{k}$, e/ $\vec{k} \times \vec{i}$

Exo 3: a/ Find all the numbers x for which $(x\vec{i} + 11\vec{j} - 3\vec{k}) \perp (2x\vec{i} - x\vec{j} - 5\vec{k})$
b/ Find all the numbers x for which the angle between $\vec{c} = x\vec{i} + \vec{j} + \vec{k}$ and $\vec{d} = \vec{i} + x\vec{j} + \vec{k}$ is $\frac{\pi}{3}$
c/ Set $\vec{a} = \vec{i} + x\vec{j} + \vec{k}$ and $\vec{b} = 2\vec{i} - \vec{j} + y\vec{k}$, compute all values of x and y for which $\vec{a} \perp \vec{b}$ and $\|\vec{a}\| = \|\vec{b}\|$

Exo 4: Find the volume of the parallelepiped with the given edges
 $\vec{a} = \vec{i} - 3\vec{j} + \vec{k}$, $\vec{b} = 2\vec{j} - \vec{k}$, $\vec{c} = \vec{i} + \vec{j} - 2\vec{k}$

Exo 5: Find $\text{comp}_{\vec{w}} \vec{v}$ and $\text{proj}_{\vec{w}} \vec{v}$ given that
 $\vec{v} = -2\vec{i} + \vec{j} + \vec{k}$ and $\vec{w} = 4\vec{i} - 3\vec{j} + \vec{k}$

Exo 6: Identify the surface

a/ $x^2 + 4y^2 - 16z^2 = 0$, b/ $5x^2 + 2y^2 - 6z^2 = 10$
c/ $x - y^2 + 2z^2 = 0$

Exo 7: Given that $\vec{f}(t) = t\vec{i} + \sqrt{t+1}\vec{j} - e^t\vec{k}$

Find : 1) the domain of $\vec{f}(t)$, 2) $\vec{f}(0)$, 3) $\vec{f}'(t)$, 4) $\vec{f}'(0)$
5) $\|\vec{f}(t)\|$, 6) $\vec{f}(t) \cdot \vec{f}'(t)$

Exo 8: Find $\vec{f}''(t)$ for $\vec{f}(t) = t \sin t \vec{i} + e^{-t} \vec{j} + t \vec{k}$

Find $\int_0^1 \vec{f}(t) dt$ for $\vec{f}(t) = t\vec{i} + \sqrt{t+1}\vec{j} - e^t\vec{k}$.

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Set #3

Q. Exo 1:

a) Find the gradient

1/ $f(x, y) = x e^{xy}$, 2/ $f(x, y) = 3x^2 - xy + y$, 3/ $f(x, y, z) = x^2 y e^{-z}$

b) Find the gradient vector at the point P

1/ $f(x, y) = 2x^2 - 3xy + 4y^2$ at $P(2, 3)$

2/ $f(x, y, z) = x - \sqrt{y^2 - z^2}$ at $P(2, -3, 4)$

Exo 2:

a) Find the directional derivative of $f(x, y) = x^2 + 3y^2$ at the point $P(1, 1)$ in the direction $\vec{i} - \vec{j}$

b) Find the directional derivative of $f(x, y, z) = (x + y^2 + z^3)^2$ at the point $P(1, -1, 1)$ in the direction of $\vec{i} + \vec{j}$

c) Find the directional derivative of $f(x, y, z) = x e^{y^2 - z^2}$ at $P(1, 2, -2)$ in the direction of the path $\vec{r}(t) = t\vec{i} + 2\cos(t-1)\vec{j} - 2e^{t-1}\vec{k}$

Exo 3: Find the rate of change of f with respect to t along the given curve

a) $f(x, y, z) = xy - yz$, $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$

b) $f(x, y, z) = x^2 + y^2$, $\vec{r}(t) = a \cos \omega t \vec{i} + b \sin \omega t \vec{j} + b \omega t \vec{k}$

2) Find $\frac{dU}{dt}$, $U = x^2 - 3xy + 2y^2$, $x = \cos t$, $y = \sin t$

3) Find $\frac{\partial U}{\partial s}$ and $\frac{\partial U}{\partial t}$, $U = x^2 - xy$ where $x = s \cos t$, $y = t \sin s$

Exo 4: Find the stationary points and the local extreme

a) $f(x, y) = 4x + 2y - x^2 + xy - y^2$, b) $f(x, y) = xy + \frac{1}{x} + \frac{1}{y}$.

c) $f(x, y) = x^3 + y^2 - 6xy + 6x + 3y - 2$.

Exo 5: Find a normal vector and a tangent vector at the $P(\sqrt{2}, 1)$

write an equation for the tangent line and the normal line

$$(x^2 + y^2)^2 = 9(x^2 - y^2)$$

Exo 6: Find an equation for the plane tangent to the surface

$xy + yz + zx = 11$ at the point $P(1, 2, 3)$

$xy + yz + zx = 1$

Set # 4

Exo 1: Compute each of the following double integrals over the indicated rectangles

a/ $\iint_R (2x - 4y^3) dx dy$, $R = [-5, 4] \times [0, 3]$

b/ $\iint_R x e^{xy} dx dy$, $R = [-1, 2] \times [0, 1]$

Exo 2: Evaluate each of the following integrals over the given region

a/ $\iint_S (4xy - y^3) dx dy$; S is the region bounded by $y = \sqrt{x}$ and $y = x^3$
 $0 \leq x \leq 1$

b/ $\iint_S e^{-\frac{y^2}{2}} dx dy$; S is the triangle formed by the lines $2y = x$, $y = 1$ and y -axis.

c/ $\iint_S dy dx$; S is the region bounded by $y + x = 5$ and $xy = 1$

Exo 3: Find the volume of the solid that lies below the surface given by $z = 16xy + 200$ and lies above the region in the xy -plane bounded by $y = x^2$ and $y = 8 - x^2$

Exo 4: Evaluate the following integral by converting it into polar coordinates

$\iint_D e^{x^2+y^2} dx dy$; D is the unit circle centered at the origin

Exo 5:

a/ Evaluate the following integral:

$\iiint_E 8xyz dx dy dz$; $E = \{(x, y, z) \mid 2 \leq x \leq 3, 1 \leq y \leq 2, 0 \leq z \leq 1\}$

b/ Using triple integration, Find the volume of the solid $\iiint dx dy dz$ bounded above by the parabolic cylinder $z = 4 - y^2$ and bounded below by elliptic paraboloid $z = x^2 + 3y^2$

c/ Evaluate $\iiint_E 16z dx dy dz$ where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$

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Set #8

22. Exo 1: Prove,

1/ If $x > 0$, then $x^{\frac{1}{n}} \rightarrow 1$ as $n \rightarrow \infty$

2/ If $|x| < 1$, then $x^n \rightarrow 0$ as $n \rightarrow \infty$

3/ $n^{\frac{1}{n}} \rightarrow 1$ as $n \rightarrow \infty$

4/ For each real x , $(1 + \frac{x}{n})^n \rightarrow e^x$ as $n \rightarrow \infty$

Exo 2: Find:

1/ $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$, 2/ $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$, 3/ $\lim_{x \rightarrow \infty} \frac{e^x}{x^2}$

4/ $\lim_{x \rightarrow -\infty} x e^x$, 5/ $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$, 6/ $\lim_{x \rightarrow 0^+} [\frac{1}{x} - \frac{1}{\sin x}]$

7/ $\lim_{x \rightarrow 0^+} (1 + 2x)^{\frac{1}{x}}$, 8/ $\lim_{x \rightarrow 0} (3x)^{4x}$

Exo 3: Check:

a/ $\int_0^{\infty} e^{-2x} dx = \frac{1}{2}$, b/ $\int_1^{\infty} \frac{dx}{x^2} = 1$, c/ $\int_1^{\infty} \frac{dx}{x}$ diverges

d/ $\int_{-\infty}^1 e^x dx = e$, e/ $\int_{-\infty}^1 \cos \pi x dx$ diverges.

Exo 4:

a/ check:

1/ $\int_0^1 (1-x)^{-2/3} dx = 3$, 2/ $\int_0^2 \frac{dx}{x}$ diverges

b/ calculate $\int_{-2}^1 \frac{dx}{x^{4/5}}$

Quadric Surfaces

In 3-dimensional space, we may consider quadratic equations in three variables x , y , and z :

$$ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + iz + j = 0$$

Such an equation defines a surface in 3D. *Quadric surfaces* are the surfaces whose equations can be, through a series of rotations and translations, put into quadratic polynomial equations of the form

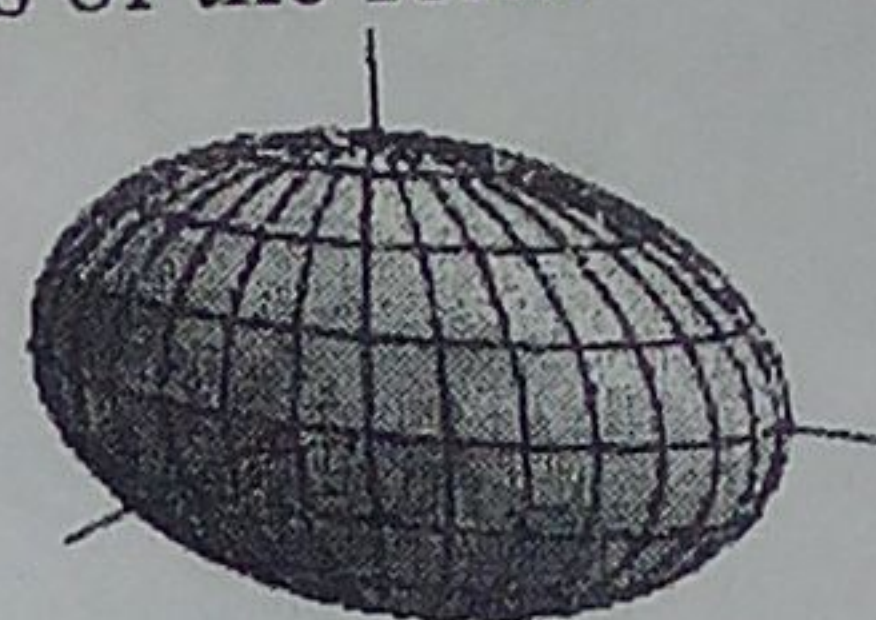
$$\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} \pm \frac{z^2}{c^2} = k \quad (1)$$

which are quadratic in at least two variables. There are distinct types of quadric surfaces, arising from different forms of equation (1).

1. Ellipsoids

The ellipsoid is the surface given by equations of the form

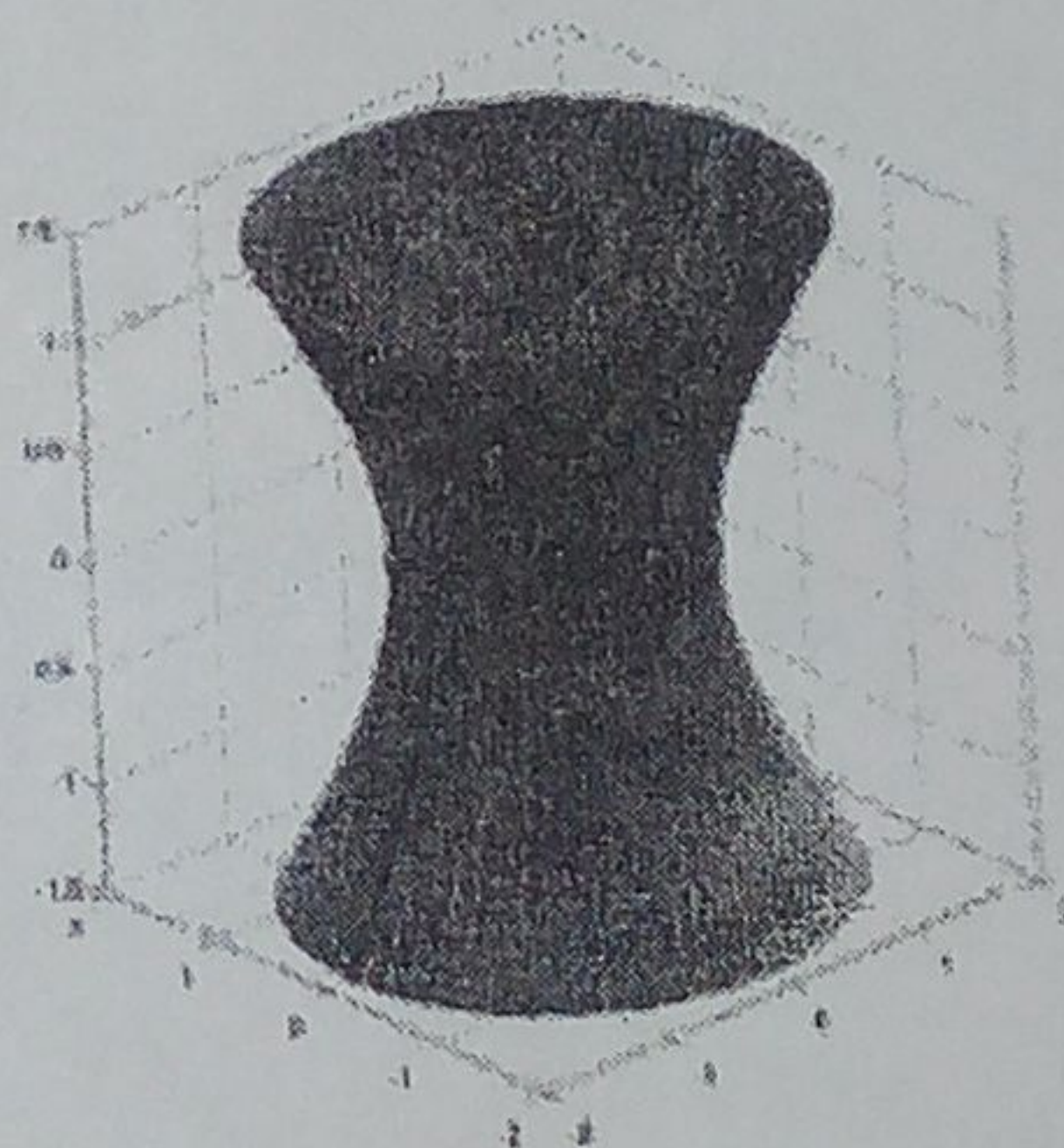
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$



2. Hyperboloid

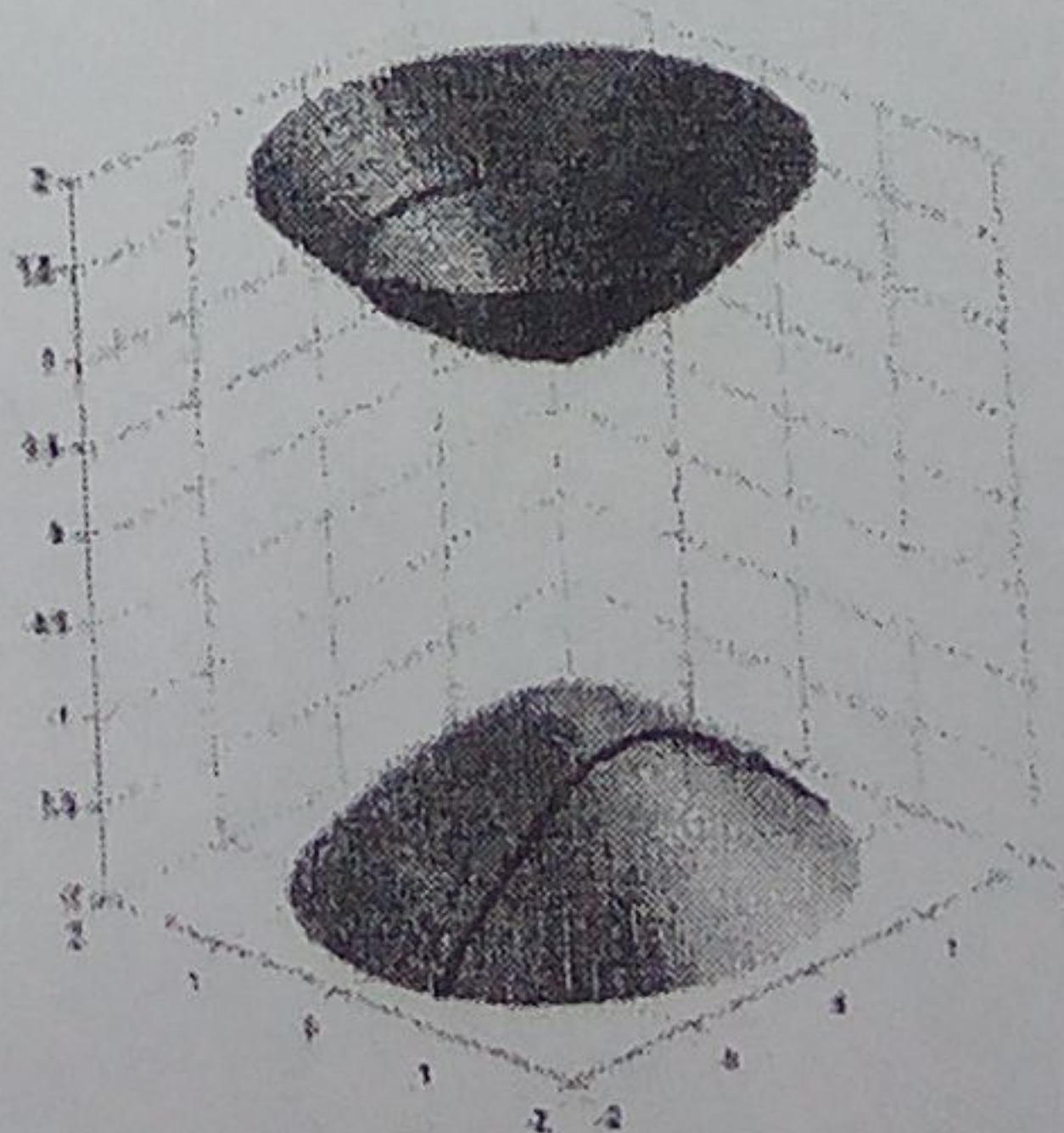
2.a Hyperboloid of one sheet

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



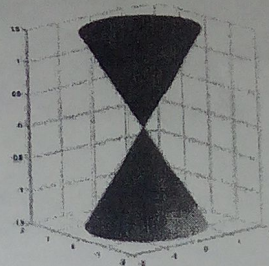
2.b Hyperboloid of two sheets

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

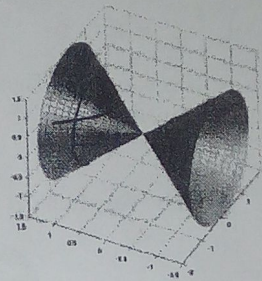


3. Cone

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$



elliptic cone

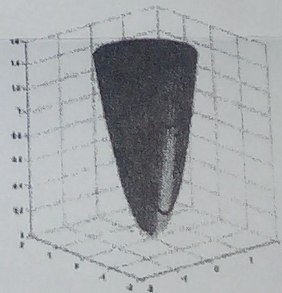


$$\frac{y^2}{b^2} = \frac{x^2}{a^2} + \frac{z^2}{c^2}$$

4. Paraboloid

4.a The elliptic Paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$



4.b The hyperbolic paraboloid

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

